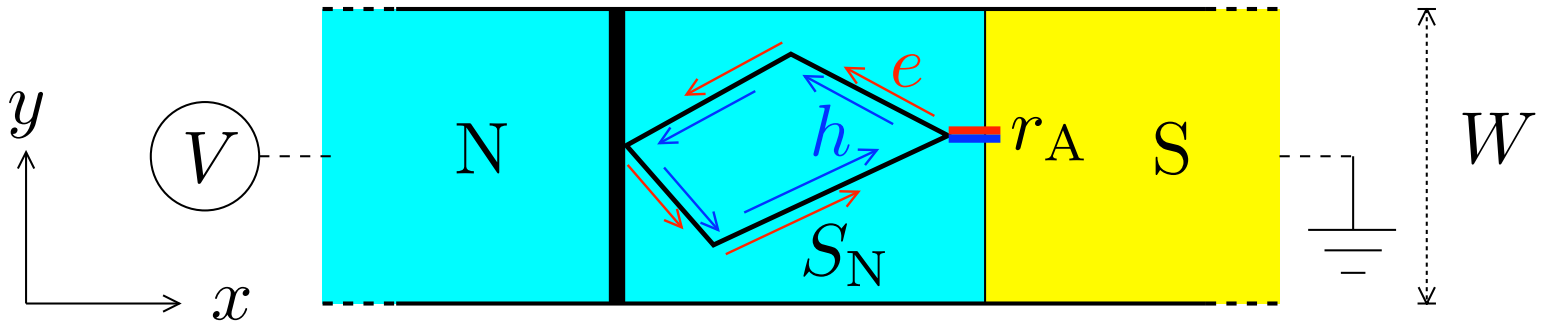
Zero-voltage conductance peak from weak antilocalization in a Majorana nanowire

D. I. Pikulin, J. P. Dahlhaus, M. Wimmer, and C. W. J. Beenakker

Arxiv:1206.6687



Outline

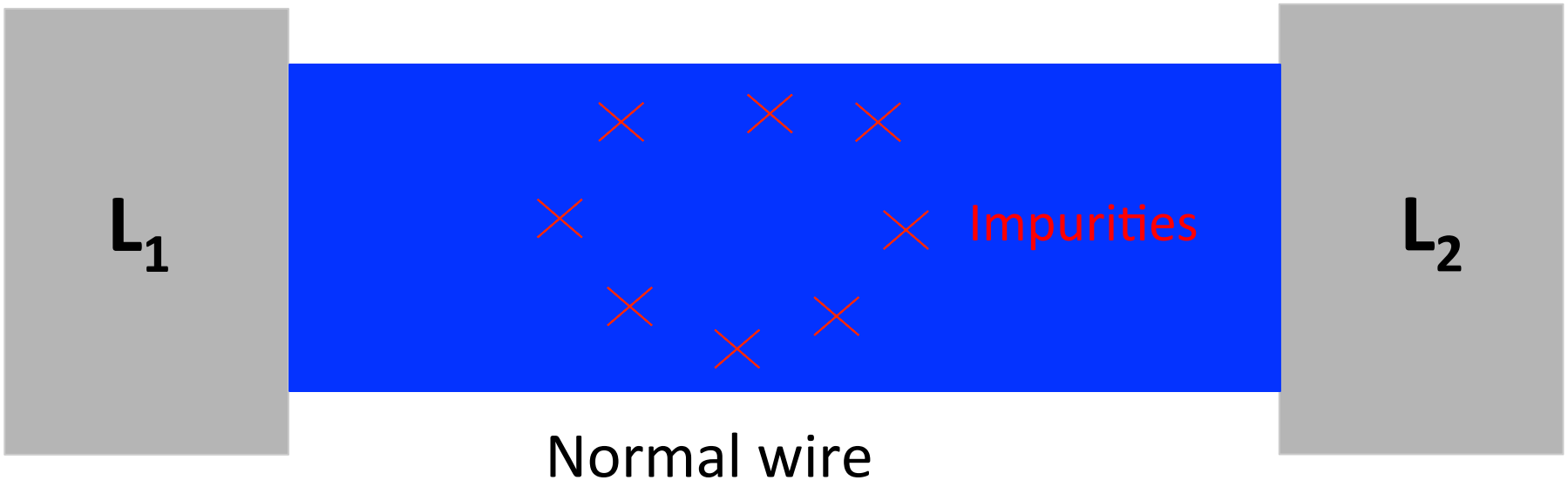
Weak (anti) localization

Scattering matrix theory

Weak antilocalization in Majorana nanowires

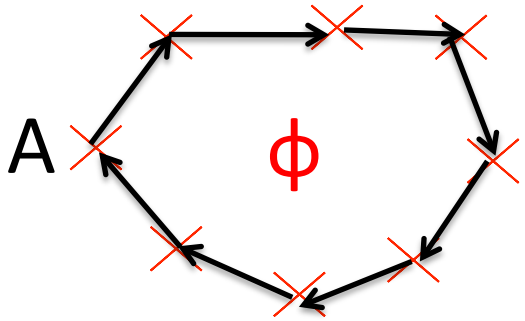
Conclusions

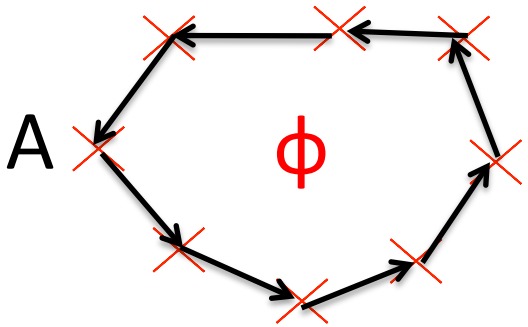
Weak (anti) localization



Normal wire

Constructive (destructive) interference between time-reversed (Feynman) paths

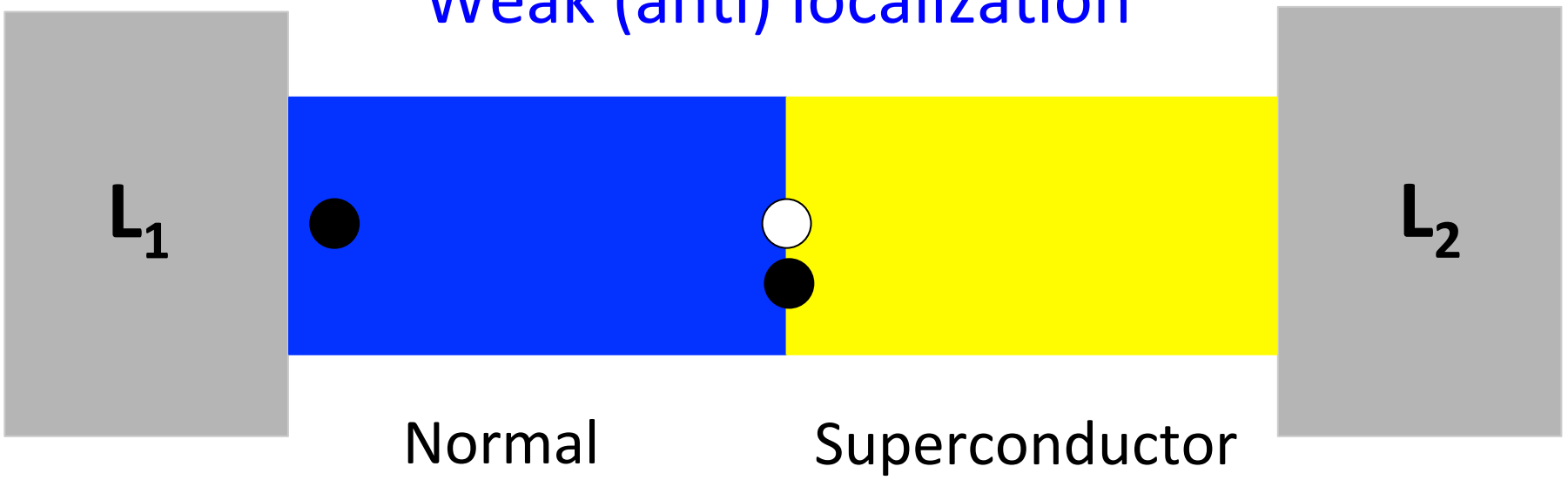




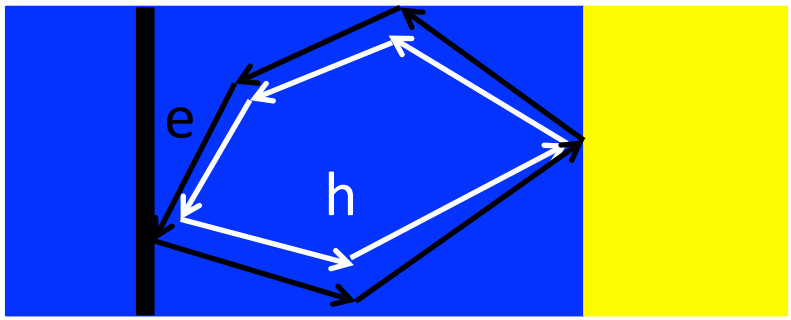
The electron wants to stay at fixed position!

Interference is destroyed by an external magnetic field

Weak (anti) localization



Andreev reflection



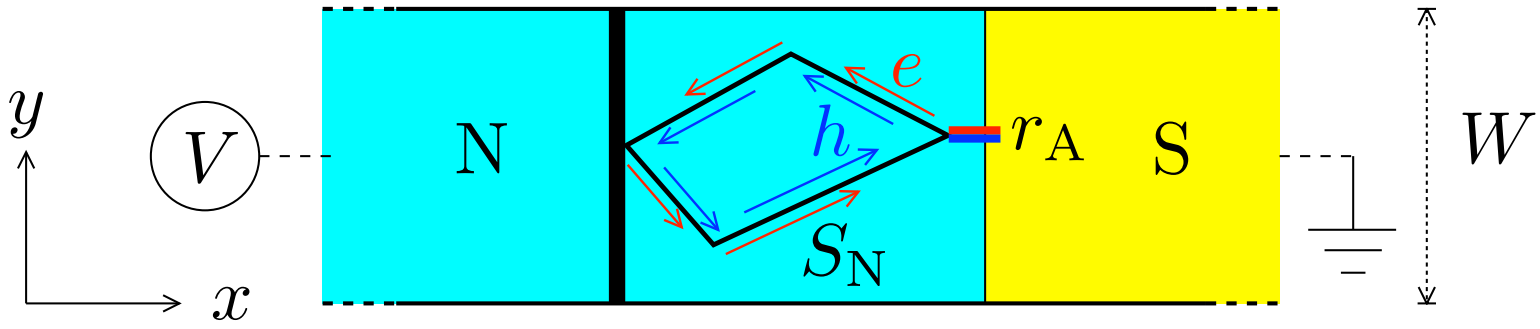
Particle-hole symmetry

Phase conjugated scattering events

Weak (anti) localization

Destroyed by finite voltage-bias

Scattering matrix theory



N propagating electronic modes through the wire in the normal state

Topological invariant **Q** counts the number of topologically protected modes at interface

No time-reversal invariance (due to magnetic field) and no spin-rotation symmetry (due to spin-orbit)

$$\Rightarrow \quad Q \in \{0, 1\}$$

Scattering matrix theory

The appropriate description of transport through mesoscopic samples is given by the scattering matrix formalism

$$S_{\mathrm{N}} = \begin{pmatrix} s_0 & 0 \\ 0 & s_0^* \end{pmatrix} \qquad s_0 = \begin{pmatrix} r' & t' \\ t & r \end{pmatrix}$$

N x N reflection matrix

N x N transmission matrix

Scattering matrix for the normal region

Scattering matrix for Andreev's process

$$r_A = \begin{pmatrix} r_{ee} & r_{eh} \\ r_{he} & r_{hh} \end{pmatrix}$$

Conductance

$$G = G_0 \mathrm{Tr}(r_{he}^\dagger r_{he}) = G_0 \sum_n R_n$$

with $G_0 = 2e^2/h$

Topological invariant Q $Q = [1 - \det(r_A)]/2$

Scattering matrix theory

$$N = 2M + 1 \qquad r_A = \begin{pmatrix} \Gamma & \Lambda \\ \Lambda^* & \Gamma \end{pmatrix}$$

$$\Lambda = \Lambda_M \oplus Q \qquad \Gamma = \Gamma_M \oplus (1 - Q)$$

Mode mixing part is incorporated in normal part S_N

PRB **83**, 085413 (2011)

$$N = 2M \quad Q = 0$$

$$\Lambda = \bigoplus_{m=1}^{M} \begin{pmatrix} 0 & -i \sin \alpha_m \\ i \sin \alpha_m & 0 \end{pmatrix} \qquad \Gamma = \bigoplus_{m=1}^{M} \begin{pmatrix} \cos \alpha_m & 0 \\ 0 & \cos \alpha_m \end{pmatrix}$$

$$N = 2M \qquad Q = 1$$

$$R_n = \sin^2 \alpha_n$$

$$\Lambda = \Lambda_{M-1} \oplus \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad \Gamma = \Gamma_{M-1} \oplus \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Scattering matrix theory

$N = 4, Q = 1$

$$\begin{pmatrix} c^{\uparrow}_{e,1} & c^{\downarrow}_{e,1} & c^{\uparrow}_{e,2} & c^{\downarrow}_{e,2} & c^{\uparrow}_{h,1} & c^{\downarrow}_{h,1} & c^{\uparrow}_{h,2} & c^{\downarrow}_{h,2} \end{pmatrix}$$

$$\begin{pmatrix}
\cos(\alpha_1) & 0 & 0 & 0 & 0 & -i\sin(\alpha_1) & 0 & 0 \\
0 & \cos(\alpha_1) & 0 & 0 & i\sin(\alpha_1) & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & i\sin(\alpha_1) & 0 & 0 & \cos(\alpha_1) & 0 & 0 & 0 \\
-i\sin(\alpha_1) & 0 & 0 & 0 & 0 & \cos(\alpha_1) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Scattering matrix theory

Polar decomposition of normal region scattering matrix S_N

$$s_0 = \underbrace{\begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix}}_{\text{Mixing of modes}} \begin{pmatrix} -\sqrt{1-\mathcal{T}} & \sqrt{\mathcal{T}} \\ \sqrt{\mathcal{T}} & \sqrt{1-\mathcal{T}} \end{pmatrix} \begin{pmatrix} U' & 0 \\ 0 & V' \end{pmatrix}$$

$$\mathcal{T} = \mathrm{diag}\,(T_1, T_2, \ldots T_N)$$

Assumptions:

- Andreev reflections with unit probability

$$R_n = \sin^2(\alpha_n) = 1 \;\; \forall \; n$$

- $N - Q = 2M$ is even

$$\Rightarrow \; r_{he} = t'^{*}\Lambda^{*}(1 - r\Lambda r^{*}\Lambda^{*})^{-1}t, \quad \Lambda = \sigma_y^{\oplus M} \oplus \mathbb{1}_Q$$

$$\Rightarrow \; \boxed{G/G_0 = \mathrm{Tr}\,\mathcal{T}\mathcal{M}\mathcal{T}\mathcal{M}^{\dagger}}$$

$$\mathcal{M} = (1 - \Omega^{*}\sqrt{1-\mathcal{T}}\Omega\sqrt{1-\mathcal{T}})^{-1}\Omega^{*}$$

$$\Omega = V'\Lambda V^{*}$$

Random scattering matrix theory with disorder

Isotropic mixing of the modes in the normal region

→ Unitary matrices U and V entering the polar decomposition of S_N are uniformly distributed in the unitary group U(N)

Assume constant $T_1=T_2=\ldots T_N=T$

→ Average conductance is calculated by integration over U(N) with th uniform (Haar) measure

$$\langle G\rangle = T^2 G_0 \int_0^{2\pi} d\phi\, \rho(\phi) \left|1-(1-T)e^{i\phi}\right|^{-2}$$

Zero-bias voltage

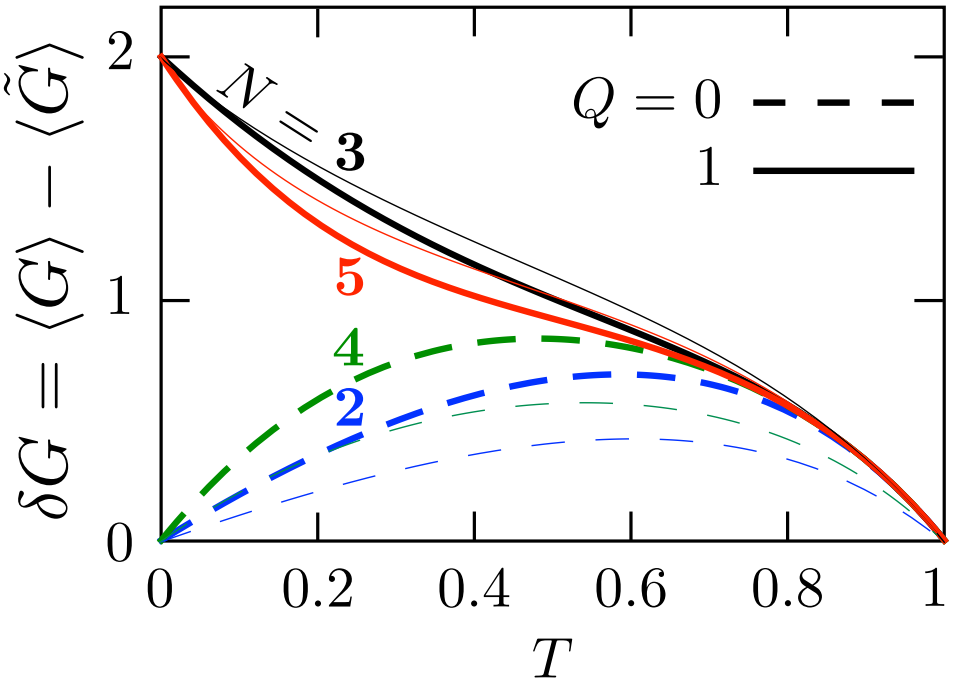
$\rho(\phi) = \langle \sum_n \delta(\phi-\phi_n)\rangle$ $e^{i\phi_n}$ eigenvalues of $\Omega^*\Omega$

Weak antilocalization in Majorana nanowire

$$\frac{\langle G\rangle_{\rm D}}{G_0}=\begin{cases}2T & \text{for } N=2,\ Q=0,\\ 1+2T^2 & \text{for } N=3,\ Q=1,\\ 2T(2-T+T^2) & \text{for } N=4,\ Q=0,\\ 1+2T^2(3-2T+T^2) & \text{for } N=5,\ Q=1.\end{cases}$$

Finite-voltage expression

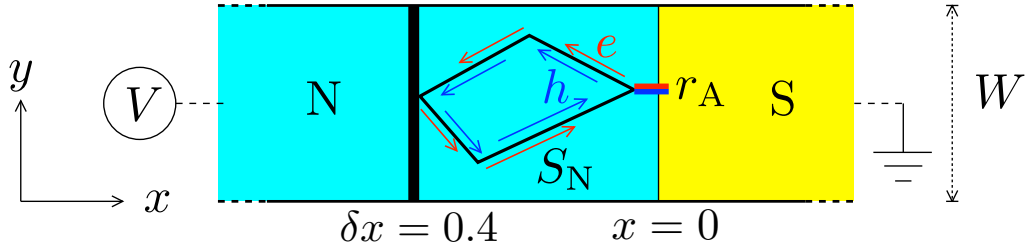
$$\langle\tilde{G}\rangle = NG_0T/(2-T)$$



$$\delta G > 0$$

Antilocalization !

Weak antilocalization in Majorana nanowire

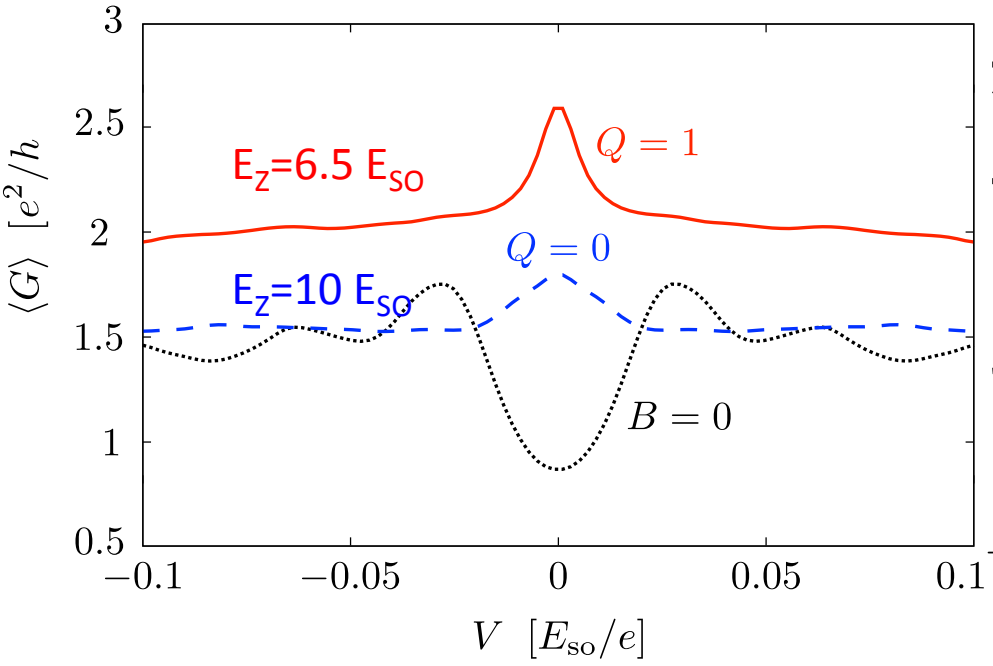


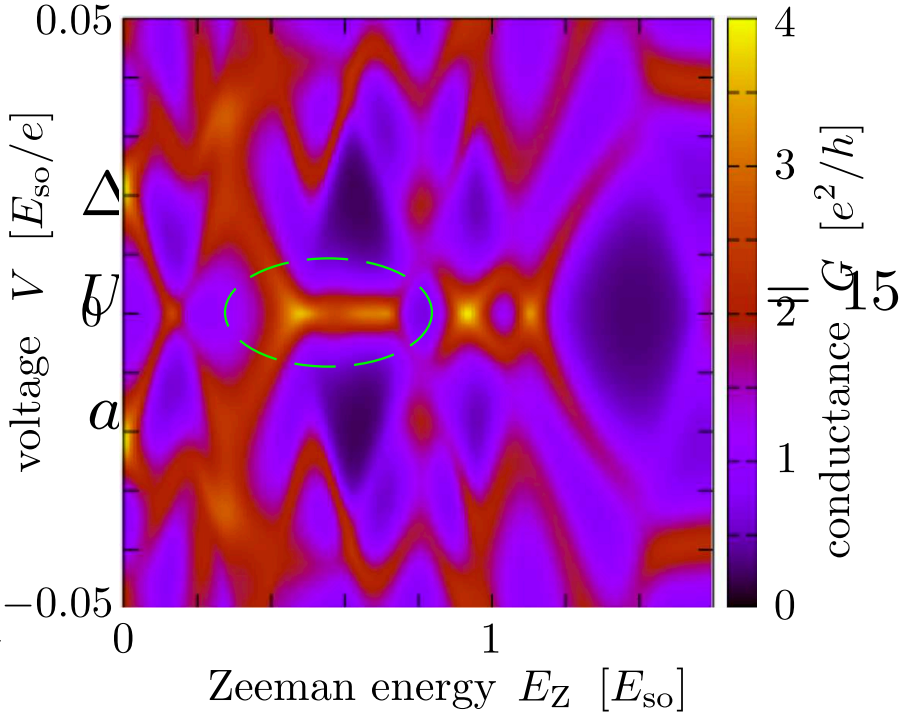
Bogoliubov-de Gennes $\mathcal{H} = \begin{pmatrix} H_0 - E_F & \Delta\sigma_y \\ \Delta^*\sigma_y & E_F - H_0^* \end{pmatrix}$ $\delta U \in (-U_{\text{disorder}}, U_{\text{disorder}})$

$$H_0 = \frac{\boldsymbol{p}^2}{2m_{\text{eff}}} + U(\boldsymbol{r}) + \frac{\alpha_{\text{so}}}{\hbar}(\sigma_x p_y - \sigma_y p_x) + E_Z\sigma_x$$

Rashba SO Zeeman

$U = U_{\text{gate}} + \delta U$ Impurity potential





Conclusions

In conclusion, we have shown that random quantum interference by disorder in a superconducting nanowire can systematically produce a zero-voltage conductance peak in the absence of time-reversal symmetry. This weak antilocalization effect relies on the same particle-hole symmetry that protects the Majorana zero-mode, but it exists in both the topologically trivial and nontrivial phase of the superconductor. A conclusive demonstration of Majorana fermions will need to rule out this alternative mechanism for a conductance peak.

Thank you for your attention!