Coulomb drag

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Introduction

Coulomb drag: induction of a drag current by momentum transfer between spatially isolated closely spaced electron systems

Measured quantity: drag resistance

\[ R_D = \frac{V_D}{I} \]

as a function of:
- temperature
- gate voltages (wire width)
- magnetic field
- ...

→ learn about electron-electron interaction in low-dimensional systems

2D: 2DEG

1D: wires in 2DEG
Some literature

- **2D systems, review article**
  
  *Electron-drag in coupled electron systems,*  

- **1D systems**
  
  - **experiments**
    
    *Experimental studies of Coulomb drag between ballistic quantum wires,*  
    
    *Negative Coulomb Drag in a One-Dimensional Wire,*  
    M. Yamamoto *et. al.* (Tarucha group Tokyo), Science **313**, 204 (2006)
  
  - **theory**
    
    *Coulomb Drag of Luttinger Liquids and Quantum Hall Edges,*  
    
    *Current Drag in Capacitively Coupled Luttinger Constrictions,*  
    
    *Coulomb Drag between One-Dimensional Conductors,*  
    
    *Coulomb drag between quantum wires,*  
  
  - **review**
    
    *Coulomb drag between ballistic one-dimensional electron systems,*  
Overview of Coulomb drag in 2D systems


• Prediction: M.B. Pogrebinskii, Sov. Phys.-Semicond. 11, 372 (1977)
  semiconductor-insulator-semiconductor layer structure, drive current $I$ in layer 1
  → drag of carriers in layer 2 due to direct Coulomb interaction
  → charge imbalance across layer 2
  → drag voltage $V_D$ induced in layer 2
  stationary state: electric field balances the frictional force of the interlayer scattering

• Expectation for single-particle Coulomb scattering

  \[ R_D \propto T^2 \]
  at low temperature, perturbation treatment of interaction scattering between states within $k_B T$ of the Fermi surface for each layer (exclusion principle)

• Additional contributions (deviations)
  - phonon exchange → enhancement for $T < 0.1 \ T_F$
  - plasmons → enhancement for $T > 0.2 \ T_F$
  - disorder → $\propto T^2 \ln T$
  - magnetic field
**Phonons and plasmons**

![Graph showing scattering rate and temperature relationship](image)

**Figure 2.** The scattering rate due to the Coulomb scattering and virtual phonons $\tau_D^{-1}/T^2$ as a function of temperature for different separations. Note that $\rho_D \propto \tau_D^{-1}$. The solid circles are the experimental results of reference [1], and the solid curves are the theoretical results from reference [24]. Inset: the contribution to $\rho_D \propto \tau_D^{-1}$ due to exchange of virtual phonons as a function of temperature. (Reproduced from reference [24].)


**phonon contribution**

- $\propto T^5$ or $T^7$ @ low $T$
- $\propto T$ @ high $T$

maximal when $k_{F,1} = k_{F,2}$ (not shown)

![Graph showing transresistivity vs. reduced temperature](image)

**Figure 3.** The scaled transresistivity $\rho T^{-2}$ ($\rho \propto \rho_D$) versus the reduced temperature for different densities (the densities in the two layers are the same). The dashed (solid) curves show the RPA (Hubbard) calculations of Flensberg and Hu [35], and the circles show the experimental results of reference [37]. (Reproduced from reference [37].)


plasmons thermally available

Enhancement due to larger phase-space for scattering
Magnetic field

Figure 4. The transresistance $R_T(\rho_D)$ as a function of magnetic field $B$ for a coupled electron gas with a separation barrier of 30 nm, shown for different temperatures (plotted with offsets for clarity). The electron density is $n = 3.2 \times 10^{11}$ cm$^{-2}$ in both layers. The longitudinal resistance is also shown. (Reproduced from reference [55].)


Twin peaks in longitudinal transresistance: predicted numerically & observed

Hall drag (transverse): predicted but never observed in the absence of interlayer tunneling

Figure 5. The measured temperature dependence of $\rho_D$ at $\nu = 1/2$ (solid curve). The broken curves are calculations from references [9, 63] of $\rho_D$ assuming two different values of the composite-fermion mass (dotted, $m^* = 12m_0$; dashed, $m^* = 4m_0$, where $m_0$ is the GaAs band mass). (Reproduced from reference [28].)


? increase with temperature is too large

? saturation at low temperature ↔ dephasing issue
Coulomb drag in 1D: The quest for Luttinger-liquid behavior

For truly one-dimensional, infinite systems in the presence of electron-electron interaction, theory predicts:

- breakdown of the Fermi-liquid description: description in terms of quasiparticles invalid

- description in terms of collective oscillations (plasmons) of the interacting electrons
  → Tomonaga-Luttinger-liquid description

Coulomb drag provides a direct way to investigate this prediction!

Questions:

- what is observed in experiments? (quasi one-dimensional, finite-length systems)

- what can theory tell for finite-length systems?
1D: Fermi-liquid based theory

- leads: ideal electronic reservoirs at thermal equilibrium
- intrawire interaction: does not result in a current variation because of the quasimomentum conservation in the electron-electron collisions
- interwire interaction: direct electron-electron collisions mediated by the Coulomb interaction

\[ \epsilon_{nk}^{(1)} + \epsilon_{n'k'}^{(2)} = \epsilon_{l,k+q}^{(1)} + \epsilon_{l',k'-q}^{(2)} \]

→ solve Boltzmann equation for temperature T low compared to the Fermi energy

1D: Fermi-liquid based theory

Ohmic transport $eV << k_B T$

$\rightarrow$ drag current

$$\frac{I_D}{I} = \frac{4e^4 m^3 L k_B T}{\pi \hbar^3 \kappa^2 N} \sum_{nn'} D_{nn'}$$

with

$$D_{nn'} = \frac{1}{(k_n^{(1)} + k_n^{(2)})^3} g_{nn'} (k_n^{(1)} + k_n^{(2)})$$

$$g_{nn'}(q) = \left| \int d^2 r_\perp \int d^2 r'_\perp |\phi_n(r_\perp)|^2 |\phi_n(r'_\perp)|^2 K_0(q |\Delta r_\perp|) \right|^2$$

$\rightarrow$ exponential decay with wire separation

$$k_n^{(1,2)} = \sqrt{2m \left[ \mu_n^{(1,2)} - \varepsilon_n^{(1,2)}(0) \right]}$$

$\mu^{(0)}$ chemical potential of wire $j$

$\varepsilon_{n}^{(0)}$ dispersion of wire $j$

Linear temperature dependence also for

$$R_D = \frac{-V_D}{I} = \frac{I_D G_D}{I}$$

Figure 3. $I_D / I$ is plotted (for $\mu^{(1)} = \mu^{(2)} = \mu$) as a function of $W_1 / W_2$ where the width of wire 1 is controlled through gate voltage ($\mu = 14$ meV, $T = 1$ K, $W_2 = 42$ nm, $L = 1 \mu$m, $\kappa = 13$ and the spacing between wires is 50 nm).

Peaks when the channel velocities of the two wires are aligned

non-Ohmic transport $eV >> k_B T$ $\rightarrow$ richer T dependence, similar features
1D: Tomonaga-Luttinger-liquid theory

- 1D, infinite interacting electron system:

\[ H = \frac{v_F}{2} \int dx \left[ \Pi(x)^2 + \frac{1}{g^2} (\partial_x \varphi(x))^2 \right] \]

solutions: \( \varphi(x,t) \) waves (plasmons) with velocity \( v = v_F/g \)

- two wires \( (k_{F,1} = k_{F,2}) \) → \( \varphi_1, \varphi_2 \) → \( \varphi_{\pm} = \varphi_1 \pm \varphi_2 \)

- interwire backscattering of electrons \( (q \approx 2k_F) \) (dominant contribution)

\[ H_C = \lambda \int dx \cos \left[ \sqrt{8 \pi} \varphi_-(x) \right] \]

Insight for very large coupling \( \lambda \)
- interlocked charge density waves
- drag mechanism

- finite-length wire → steplike \( g(x) \) (inhomogeneous Luttinger liquid)

B. V. Ponomarenko, PRB 52, R8666 (1995)
1D: Tomonaga-Luttinger-liquid theory

- Equal lengths

Yu. V. Nazarov and D. V. Averin, PRL 1998

FIG. 2. Induced voltage difference $\delta V$ in the perturbative regime for constrictions with $k_{F1} = k_{F2}$, normalized to $V_0(g) = J^2 u\hbar E_F^{-2}(u/E_F L)^{1/2}$. The curves $a, b, c, d, e$ correspond to $g = 0.1, 0.3, 0.5, 0.6, 1.0$, respectively.

- Different lengths

1D: Tomonaga-Luttinger-liquid theory

- Temperature dependence
  - linear drag ($V \to 0$), renormalization analysis

  \[ R_D \propto \left( \frac{T}{\omega_c} \right)^{4g-3} \quad \text{(spinless)}, \quad R_D \propto \left( \frac{T}{\omega_c} \right)^{2g-1} \quad \text{(spinful)} \]

  \[ T \gg M, \ T_L : \quad R_D \propto T \]

  \[ T < M < T_L : \quad R_D \propto T, \ T < M < T_L : \quad R_D \propto T^2 \]

- non-linear drag, numerical simulations

\[ I / I_0 \]

\[ \frac{V}{V_0} \]

\[ \frac{T}{T_L} \]

1D: Experiments

- Experimental challenge

  - create parallel, electrically isolated 1D wires
    with separation large enough → interwire tunneling suppressed
    but small enough → drag voltage of reasonable magnitude

  - drag voltage: small magnitude, must be distinguished from spurious signals

- Realizations

  n-AlGaAs/GaAs heterostructure (2DEG) with Schottky gates

1D: Ballistic transport and interwire tunneling

- Ballistic transport

no peaks in pinch-off / dips in first plateau
→ ballistic transport

faded plateaus, sharper with magnetic field
→ non-adiabaticity @ constriction opening
and scattering @ wire edges

- Suppression of interwire tunneling

Figure 2. The conductance staircase of the top wire as a function of the bias voltage $V_g$ applied to the gates $T$ and $M$ with gate $B$ grounded at 60 mK and in different magnetic fields: 0 (1), 0.35 (2), and 0.86 T (3). Similar results were obtained for the bottom wire except for a small difference in the pinch-off voltage.

Figure 3. Intermesh tunneling current $I_{MN}$ as a function of the middle gate voltage $V_M$ at 60 mK and in zero magnetic field. $I$ is the drive current in the top wire. $V_M = -1.5$ V, $V_T = -1.2$ V, and $V_{D,M} = 300 \mu$V. Note that the above values of $V_B$ and $V_T$ are approximately those at which a maximum drag effect is observed in later measurements with $V_M = -0.74$ V.

**1D: drag voltage and interwire separation**

- Drag voltage shows strongly peaked structure

![Graph showing drag voltage and current relationship](image)

- Exponential dependence on interwire separation

![Graph showing exponential relationship](image)


> vary drive wire (top) gate voltage → change drive current

- Drag voltage peaks aligned with drive current risings

![Graph showing peak alignment](image)

> All compatible with both Fermi liquid and Tomonaga-Luttinger liquid theories

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**Figure 4.** Drag voltage $V_D$ and drive current $I$ as functions of the top gate voltage $V_T$ at 70 mK and in zero magnetic field with a drive voltage of 300 $\mu$V. $V_M = -0.74$ V, $V_B = -1.525$ V. For these values of $V_M$ and $V_B$, the Fermi level $E_F$ is located just above the bottom of the lowest 1D subband of the drag wire.

**Figure 8.** The dependence of the drag response on the interwire separation. (a) The maximum $V_D^{MAX}$ of the first drag peak of figure 6, in a magnetic field, as a function of the middle gate voltage $V_M$ at 60 mK. (b) The natural logarithm of the corresponding drag resistance $R_D$ as a function of $V_M$. The dotted line is a linear fit to the data points.

1D: temperature dependence

- Non-linear, power-law temperature dependence

**Figure 9.** The dependence of the drag response on the temperature. (a) The drag voltage $V_D$ as a function of the top gate voltage $V_T$ in zero magnetic field with 300 μV drive voltage at temperatures 70, 180, 300, 450, and 900 mK, corresponding to the curves in the order of decreasing peak height.

Non-linearity cannot be attributed to a temperature dependence of the conductance $R_D = \frac{I_D G_D}{I}$.

- Fermi liquid: $R_D \propto T$
- Luttinger liquid: $R_D \propto T^{2g-1}$

**Figure 10.** The temperature dependence of the drag resistance $R_D$ corresponding to $V_D^{MAX}$ for the first drag peak of figure 9 in zero field (a) and in a magnetic field of 0.86 T (b). Note that the data points at the low end of the temperature range fall below the power-law curve indicating a suppression of the drag effect at these temperatures.

1D: temperature dependence

- Non-linear, power-law temperature dependence

Figure 9. The dependence of the drag response on the temperature. (a) The drag voltage $V_D$ as a function of the top gate voltage $V_T$ in zero magnetic field with 300 $\mu$V drive voltage at temperatures 70, 180, 300, 450, and 900 mK, corresponding to the curves in the order of decreasing peak height.
1D: Negative Coulomb drag and wire asymmetry

- Negative Coulomb drag **@ low electron density, low temperature, high magnetic field**

**Fig. 2.** (A) $R_d$ versus $V_{g_{\text{drive}}}$ of a CWO sample with $L_c = 4 \, \mu m$ measured for $V_{g_{\text{center}}} = -0.9 \, V$, $V_{g_{\text{drive}}} = -0.9 \, V$, and $I = 1 \, nA$ at $T = 10 \, mK$. The black, red, green, blue, and light blue lines are the data for $B$ values of 0.9, 1.1, 1.3, 1.5, and 1.7 T, respectively. Negative drag was observed for $V_{g_{\text{drive}}} < -1.0 \, V$ in a magnetic field of 1.3 T and for $V_{g_{\text{drive}}} < -0.8 \, V$ in 1.7 T. (B) Drag resistance versus $V_{g_{\text{drive}}}$ measured at a magnetic field of 7 T for $V_{g_{\text{center}}} = -0.9 \, V$, $V_{g_{\text{drive}}} = -0.8 \, V$, and $I = 1 \, nA$. Conductance of the drive wire was well below the first spin-resolved plateau but not very close to the pinch-off. The negative drag became small as the temperature was raised from 200 mK (black) to 400 mK (red), 600 mK (green), and 800 mK (blue).

- Wire asymmetry

**Fig. 3.** $R_d$ versus $V_{g_{\text{drag}}}$ of a CWO sample with $L_c = 2 \, \mu m$ and $L_l = 4 \, \mu m$ measured at 10 T for $V_{g_{\text{center}}} = -0.95 \, V$ and $I = 1 \, nA$ at $T = 50 \, mK$. (A) The longer wire was used as a drive wire. Negative drag was observed for $V_{g_{\text{drive}}} < -0.60 \, V$. (B) The shorter wire was used as a drive wire. No signature of negative drag was observed in the whole $V_{g_{\text{drag}}}$ range.

M. Yamamoto et al., Science 313, 204 (2006)
Conclusions and Outlook

• 2D systems
  - well understood within Fermi liquid theory
  - open questions remain in the quantum Hall regime (in 1999...)

• 1D systems
  - experimental temperature dependence seems incompatible with Fermi-liquid theory
  - further analysis needed for the Tomonaga-Luttinger-liquid theory → in preparation!
  - negative Coulomb drag and wire asymmetry: not explained!
  - negative Coulomb drag in coupled wire and quantum dot system → M. Shimizu, ..., S. Tarucha, Physica E 26, 460 (2005)